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A Linear Phase Modulator for a Short-Hop Microwave Radio System

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A linear phase modulator is a useful component in a short-hop radio system using digital modulation. Such a modulator has been designed, built, and tested for 4-level operation at a carrier frequency of 300 MHz and for a line rate of 20 megabits.

In this paper, the design and the performance of the modulator are presented. Measurements on the phase modulator show that the performance is in agreement with the theory.

I. INTRODUCTION

A linear phase modulator is a useful component in a short-hop radio system using digital modulation.[1,2] A possible application of this modulator in a digital radio system is shown in a block diagram in Fig. 1. A shared delta modulator multiplex operating at a line rate of 20 megabits is an example of a digital multiplex which could be used in such a system.[3] The output binary signal of this multiplex can be converted into a baseband pulse sequence by a 4-level block coder.[4]

The modulator described in this paper satisfies the requirements for the above application. It is based upon the original Armstrong circuit, which is well suited to large baseband bandwidths and is reasonably linear for low modulation indexes.[5] An analysis of distortion for the type of baseband signal used in this application is discussed by C. L. Ruthroff and W. F. Bodtmann in Ref. 1.

The output of the phase modulator is at an IF frequency of 300 MHz. It can be converted in a linear mixer to any desired RF frequency in the microwave or millimeter-wave range, and can be amplified for transmission by an injection-locked oscillator amplifier.[6]

II. DESCRIPTION OF THE MODULATOR

A block diagram of the modulator is shown in Fig. 2. A quartz crystal oscillator provides a 300-MHz stable carrier frequency. The baseband

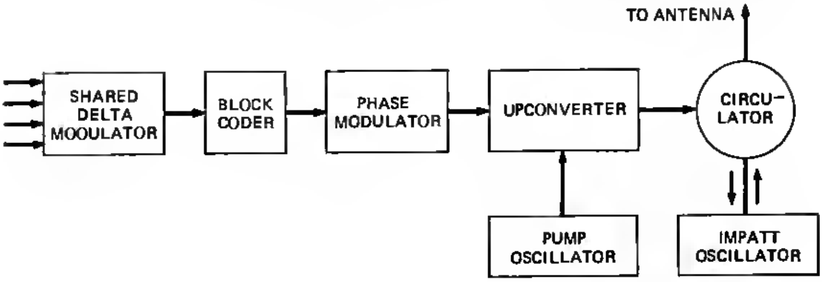

Fig. 1—Block diagram of a solid state transmitter for a digital radio system.

signal is modulated in a double-sideband suppressed-carrier amplitude modulator. At the output of this modulator another carrier, 90 percent out of phase with the first, is added to the sidebands. The combined low-index phase-modulated signal then passes through an amplifier and a times-four frequency multiplier. The output signal from the times-four multiplier is converted in a double-sideband balanced mixer to the original carrier frequency of 300 MHz as described in Ref. 1. The unwanted output frequencies from the mixer are eliminated by a lowpass filter, and the required phase-modulated signal is obtained. A limiter is not used as indicated in Ref. 1 because sufficient amplitude compression occurs in the harmonic generator.

2.1 *Double-Sideband Suppressed-Carrier Amplitude Modulator*

Carrier suppression by a double-sideband balanced mixer is a well known technique.[7] By using a conventional mixer, an isolation of about 30 dB between any two ports can be obtained. The spectrum of the

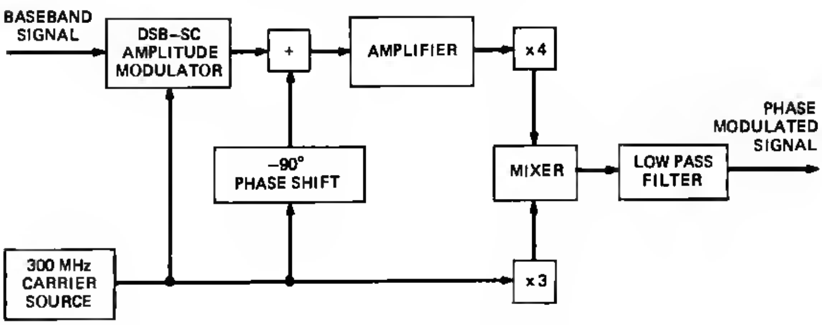

Fig. 2—Block diagram of linear phase modulator.

double-sideband suppressed-carrier amplitude-modulated signal with a modulation index of 0.2 is shown in Fig. 3. The suppression of the carrier is not sufficient, and could cause distortion, particularly if the quadrature carrier is to be reintroduced at the receiving terminal.[8] This can be explained best by considering the phasor diagrams of Fig. 4. Figure 4a shows the ideal case, in which the carrier is totally suppressed and the quadrature carrier is added without causing any distortion. Figure 4b shows a case in which the carrier is not sufficiently suppressed, and the remaining component of the carrier changes the phase of the quadrature carrier resulting in a phase error. To suppress the remaining component of the carrier, a signal of equal amplitude and 180 degrees out of phase with the carrier is added to the output signal of the mixer. This is illustrated in a block diagram in Fig. 5. The carrier from the local oscillator is divided into two parts, one of which goes to the mixer, and the other, after undergoing the required changes in amplitude and phase, is added to the output signal from the mixer. The spectrum of the resultant signal from the adder is shown in Fig. 6. Note that the carrier is suppressed by 58 dB. The carrier suppression is insensitive to changes in oscillator level; a change in the amplitude level of the crystal oscillator signal by 1 dB results in a change in the amplitude level of the carrier by a fraction of a decibel.

2.2 *Frequency Multipliers*

2.2.1 *Times-Four Multiplier*

A maximum peak deviation of $\pm 3\pi/4$ radians is required in a 4-level PSK system. If the modulator output is multiplied by four, the required peak deviation in the modulator before multiplication is then $3\pi/16$

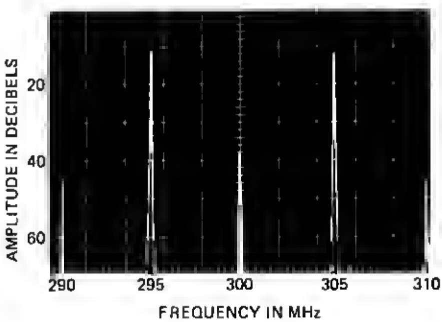

Fig. 3—Carrier suppression of a double-sideband balanced mixer.

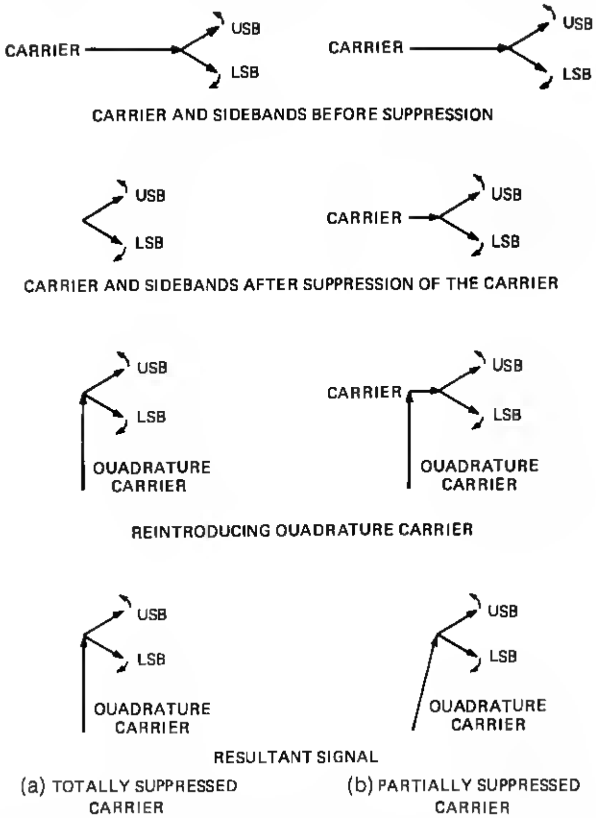

Fig. 4—Phasor diagram of the phase-modulated signal. (a) Totally suppressed carrier. (b) Partially suppressed carrier.

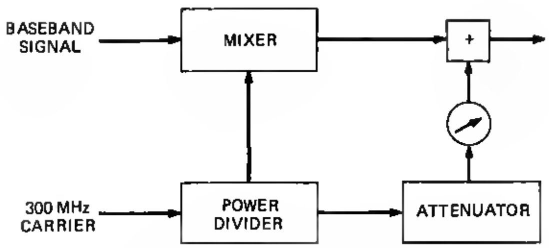

Fig. 5—Block diagram of double-sideband suppressed-carrier amplitude modulator.

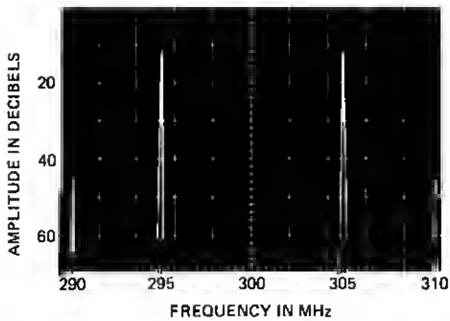

Fig. 6—Carrier suppression by improved double-sideband suppressed-carrier amplitude modulator.

radians. To obtain this magnitude of deviation, a resistive multiplier circuit is used. The conventionally designed circuit employs two Schottky barrier diodes. The input and the output matching sections are five-element 0.1-dB-ripple Tschebyscheff filters.[9] Variable air trimmer capacitors and hand-wound coils of No. 14 bare copper wire are the elements of the filters and the idler networks.

The performance of the multiplier is measured by applying the phase-modulated sine-wave with a carrier frequency of 300 MHz. The frequency of the baseband signal is varied from 5 MHz to 25 MHz in 5-MHz steps. The peak phase deviation is kept constant at 0.1 radian. The output spectrum of the multiplier corresponding to each baseband signal frequency is recorded on the same photograph, which is shown in Fig. 7. Ideally, the conversion loss of the multiplier should be the same for each baseband signal frequency. The frequency response of the

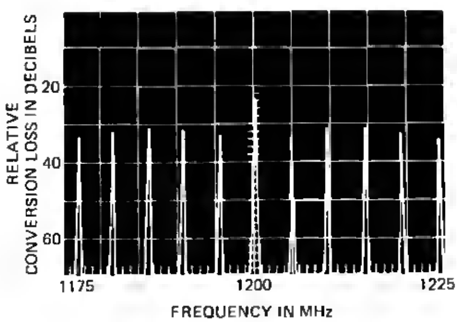

Fig. 7—Frequency response of the times-four multiplier.

mixer used in the double-sideband suppressed-carrier amplitude modulator is not flat for these baseband signal frequencies; this is shown in Fig. 8. The frequency response of the mixer is reflected in the output of the multiplier shown in Fig. 7.

2.2.2 *Times-Three Multiplier*

A times-three multiplier is used to convert the output carrier frequency of the times-four multiplier to the frequency of the carrier source as shown in Fig. 2. This multiplier need not have a broad bandwidth. The technique used for the design and fabrication of the times-three multiplier is the same as that used for the times-four multiplier.

2.3 *90-Degree Phase Shifter*

An accurate 90-degree phase shifter is made from twisted wire distributed elements using the method described in Ref. 10. The coupler, which has a crossover frequency of 300 MHz, is made of twisted pairs of FORMEX wire separated from the ground plane by polyethylene dielectric.

2.4 *Lowpass Filter*

A three-element 0.1-dB-ripple Tschebyscheff filter is used to eliminate the unwanted output frequencies from the mixer. Two variable air trimmer capacitors and a hand-wound coil of No. 14 copper wire are the elements of this lowpass filter.

III. MEASURED PERFORMANCE

The performance of the phase modulator was measured for sine-wave and square-wave baseband signals with various output phase deviations from 0.1 to $3\pi/4$ radians. For any specific input phase deviation, the

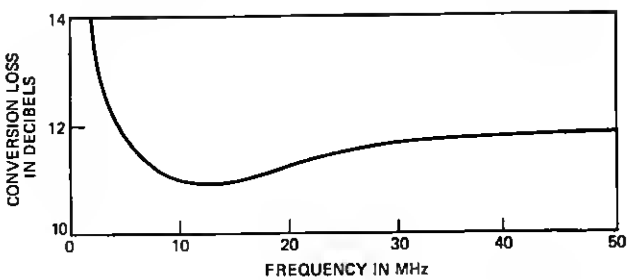

Fig. 8—Frequency response of the mixer.

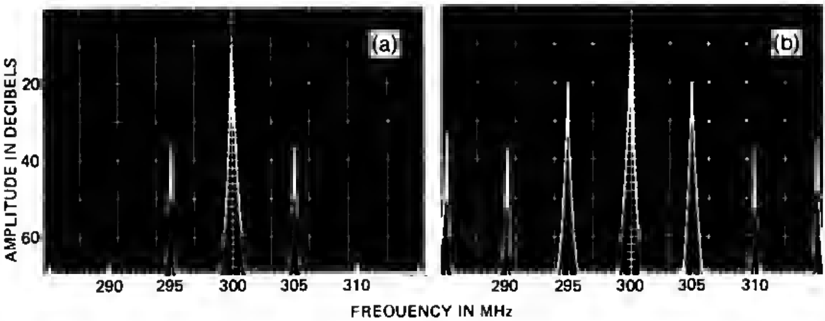

Fig. 9—Output spectrum of the phase-modulated signal from the adder. (a) 0.1 radian phase deviation. (b) $3\pi/16$ radian phase deviation.

amplitude levels of the carrier and the sidebands of the phase-modulated signal can be computed (see Appendix A); the spectrum of the output signal from the adder can then be inspected to verify the input phase deviation as shown in Fig. 9. When the signal passes through the times-four multiplier, the required output phase deviation is obtained. The output spectra of the modulator for a 5-MHz baseband signal with output phase deviations of 0.4 and $3\pi/4$ radians are shown in Fig. 10.

The detected phase-modulated signal can be compared with the input baseband signal by using a phase detector as shown in a block diagram of Fig. 11. The output signal of the detector is calculated as described in Appendix B; Figure 12 shows the calculated results for sinusoidal baseband signal with the phase deviation of $\pi/4$ and $3\pi/4$ radians. The measured results for 5-MHz and 20-MHz baseband signals with 0.4 and $3\pi/4$ radians phase deviation are shown in Fig. 13.

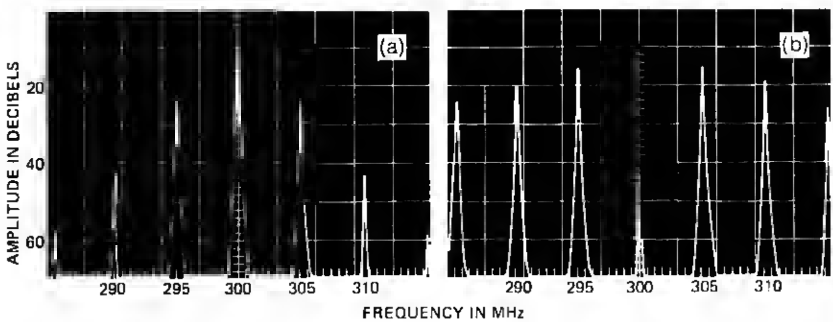

Fig. 10—Output spectrum of the phase modulator for 5 MHz sinusoidal baseband signal. (a) 0.4 radian phase deviation. (b) $3\pi/4$ radians phase deviation.

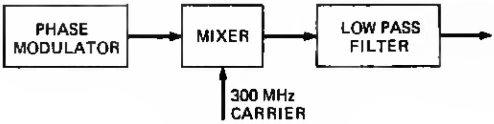

Fig. 11—Block diagram of the phase detector.

IV. CONCLUSIONS

Comparing the calculated and the measured results, particularly of Figs. 12b and 13b, it is clear that the phase modulator is performing as expected. Further, it has been demonstrated that this phase modulator is suitable for a line rate of 20 megabits. The carrier frequency is stable as it is derived from a quartz crystal oscillator. All these aspects make this phase modulator a useful component in a short-hop radio system—especially in coherent phase-shift-keyed PCM systems.

V. ACKNOWLEDGMENT

The author would like to express his thanks to C. L. Ruthroff for his helpful suggestions.

APPENDIX A

The expression for a carrier which is phase-modulated by a sinusoidal signal can be written in the general form:

$$M(t) = A_c \cos (\omega_c t + X_1 \cos \omega_m t). \tag{1}$$

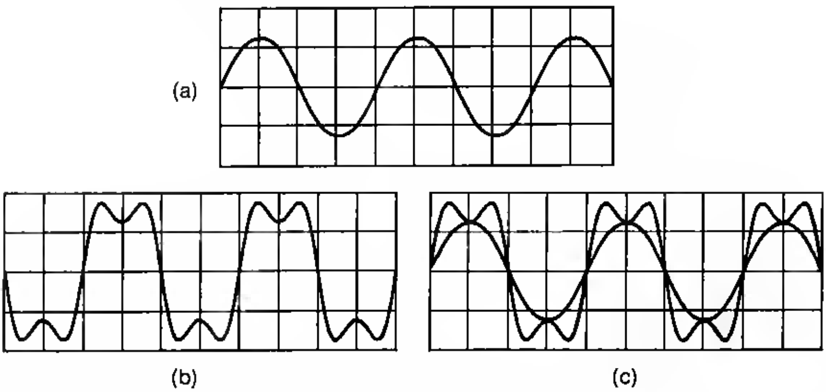

Fig. 12—Calculated output signal from the phase detector. (a) $\pi/4$ radian phase deviation. (b) $3\pi/4$ radians phase deviations. (c) Curve (a) is imposed on curve (b).

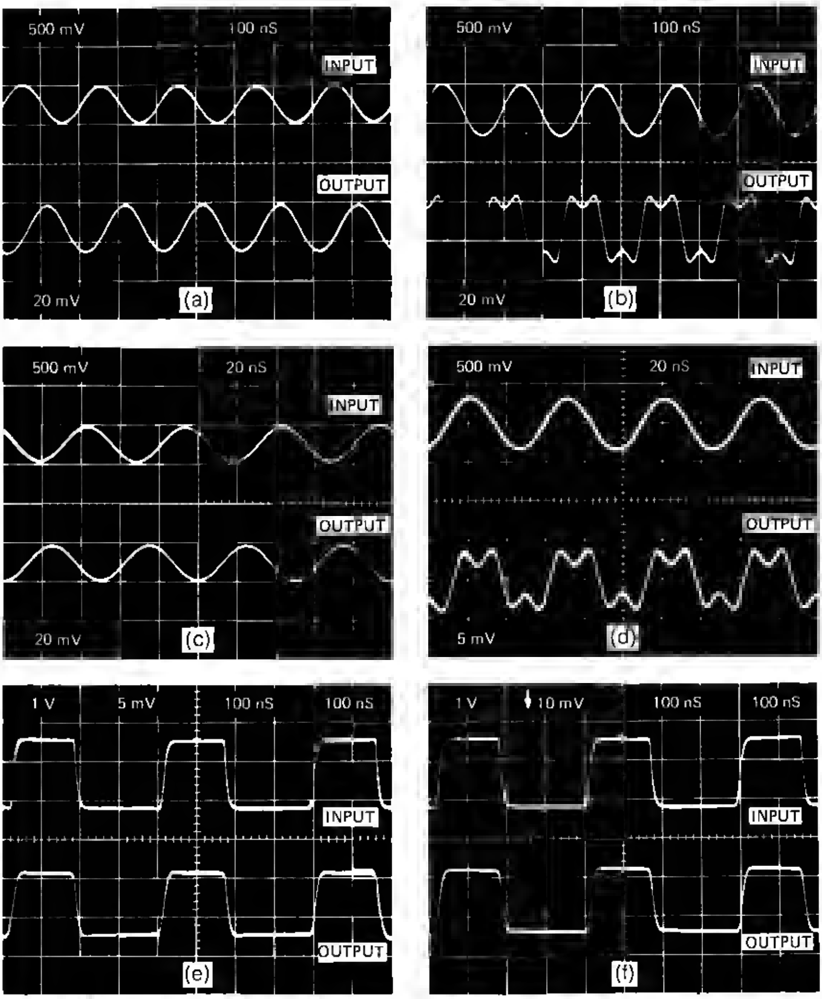

Fig. 13—Photographs of the input baseband signal and the corresponding output signal from the phase detector. (a) 0.4 radian phase deviation. (b) $3\pi/4$ radians phase deviation. (c) 0.4 radian phase deviation. (d) $3\pi/4$ radians phase deviation. (e) 0.4 radians phase deviation. (f) $3\pi/4$ radians phase deviation.

Here, X_1 is the peak phase deviation in radians. Now

$$M(t) = A_c\Big\{J_0(X_1)\cos\omega_c t + J_1(X_1)\cos\left[(\omega_c+\omega_m)t+\frac{\pi}{2}\right] + J_1(X_1)\cos\left[(\omega_c-\omega_m)t+\frac{\pi}{2}\right] - J_2(X_1)\cos(\omega_c+2\omega_m)t + J_2(X_1)\cos(\omega_c-2\omega_m)t+\cdots\Big\}. \tag{2}$$

Equation (2) shows that the magnitudes of the sidebands, relative to the carrier, can be determined by the Bessel coefficients.[5,8]

For a peak phase deviation of 0.1 radian,

$$J_0(X_1) = 0.9975$$

$$J_1(X_1) = 0.04994$$

$$J_2(X_1) = 0.00125$$

$$\frac{J_0(X_1)}{J_1(X_1)} = 19.97396 \quad \frac{J_0(X_1)}{J_2(X_1)} = 798.$$

Converting in decibels,

$$20\log\left(\frac{J_0(X_1)}{J_1(X_1)}\right) = 26\text{ dB}$$

$$20\log\left(\frac{J_0(X_1)}{J_2(X_1)}\right) = 58\text{ dB}.$$

In other words, the difference in the energy level between tbe carrier and the first sidebands is 26 dB, and between the carrier and tbe second sidebands is 58 dB for 0.1 radian peak phase deviation. This is shown in Fig. 9a.

Similarly, for a peak phase deviation of $3\pi/16$ radians,

$$J_0(X_1) = 0.9149,\ J_1(X_1) = 0.2823,\ J_2(X_1) = 0.04226$$

and

$$20\log\left(\frac{J_0(X_1)}{J_1(X_1)}\right) = 10.2\text{ dB}$$

$$20\log\left(\frac{J_0(X_1)}{J_2(X_1)}\right) = 26.65\text{ dB}.$$

Note that the difference in the energy level between the carrier and the first sidebands is 10.2 dB, and between the carrier and the second

sidebands is 26.65 dB as shown in Fig. 9b. Values of $J_n(x)$ are obtained from Ref. 11.

For the phase deviations of 0.1, 0.4, $3\pi/16$, and $3\pi/4$ radians, the calculated difference in the energy level between the carrier and the sidebands are shown in Table I.

TABLE I—THE CALCULATED DIFFERENCE IN THE ENERGY LEVEL BETWEEN THE CARRIER AND THE SIDEBANDS FOR VARIOUS PHASE DEVIATIONS

Phase deviation in radians	Difference in the energy level between the carrier and the first sidebands in dB	Difference in the energy level between the carrier and the second sidebands in dB
0.1	26	58
0.4	13.8	33.7
$3\pi/16$	10.2	26.7
$3\pi/4$	−26.3	−24.3

APPENDIX B

The phase-modulated signal can be expressed by

$$e_p = \sin\,[\omega_c t + X_1 \cos \omega_m t]$$

where

$$\omega_c = 2\pi f_c\,, \quad f_c = \text{carrier frequency}$$

$$\omega_m = 2\pi f_m\,, \quad f_m = \text{baseband signal frequency}$$

and

X_1 is the peak phase deviation in radians.

In a phase detector, e_p is multiplied by $\cos \omega_c t$, and the low-frequency part of the output is

$$e_o = \sin\,(X_1 \cos \omega_m t).$$

The values of e_o are calculated with respect to t, and are plotted for the following cases

(*i*) $X_1 = \pi/4$, $f_c = 300$ MHz and $f_m = 5$ MHz
(*ii*) $X_1 = 3\pi/4$, $f_c = 300$ MHz and $f_m = 5$ MHz.

Figures 12a and b show the curves plotted for case (*i*) and (*ii*) respectively. For comparison, the curve of case (*i*) is imposed on the curve of case (*ii*), which is shown in Fig. 12c.

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